**A Level Maths Transition Booklet**

**These are the basic Maths Skills required to succeed at A Level Maths. This booklet must be completed before you start A Level Maths.**

**Name:………………………………………………………………………**

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1. **Expanding brackets and simplifying expressions**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
* When you expand two linear expressions, each with two terms of the form *ax* + *b*, where *a*≠ 0 and *b*≠ 0, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

**Example 1** Expand 4(3*x* − 2)

|  |  |
| --- | --- |
| 4(3*x* − 2) = 12*x* − 8 | Multiply everything inside the bracket by the 4 outside the bracket |

**Example 2** Expand and simplify 3(*x* + 5) − 4(2*x* + 3)

|  |  |
| --- | --- |
| 3(*x* + 5) − 4(2*x* + 3)  = 3*x* + 15 − 8*x* – 12  = 3 − 5*x* | **1** Expand each set of brackets separately by multiplying (*x* + 5) by 3 and (2*x* + 3) by −4  **2** Simplify by collecting like terms: 3*x*− 8*x*= −5*x* and 15 − 12 = 3 |

**Example 3** Expand and simplify (*x* + 3)(*x* + 2)

|  |  |
| --- | --- |
| (*x* + 3)(*x* + 2)  = *x*(*x* + 2) + 3(*x* + 2)  = *x*2 + 2*x* + 3*x* + 6  = *x*2 + 5*x* + 6 | **1** Expand the brackets by multiplying (*x* + 2) by *x* and (*x* + 2) by 3  **2** Simplify by collecting like terms: 2*x*+ 3*x* = 5*x* |

**Example 4** Expand and simplify (*x* − 5)(2*x* + 3)

|  |  |
| --- | --- |
| (*x* − 5)(2*x* + 3)  = *x*(2*x* + 3) − 5(2*x* + 3)  = 2*x*2 + 3*x* − 10*x* − 15  = 2*x*2 − 7*x* − 15 | **1** Expand the brackets by multiplying (2*x* + 3) by *x* and (2*x* + 3) by −5  **2** Simplify by collecting like terms: 3*x*− 10*x* = −7*x* |

Practice

**1** Expand.

**Watch out!**

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is ‘+’; if the signs are different the answer is ‘–’.

**a** 3(2*x* − 1) **b** −2(5*pq* + 4*q*2)

**c** −(3*xy* − 2*y*2)

**2** Expand and simplify.

**a** 7(3*x* + 5) + 6(2*x* – 8) **b** 8(5*p* – 2) – 3(4*p* + 9)

**c** 9(3*s* + 1) –5(6*s* – 10) **d** 2(4*x* – 3) – (3*x* + 5)

**3** Expand.

**a** 3*x*(4*x* + 8) **b** 4*k*(5*k*2 – 12)

**c** –2*h*(6*h*2 + 11*h* – 5) **d** –3*s*(4*s*2 – 7*s* + 2)

**4** Expand and simplify.

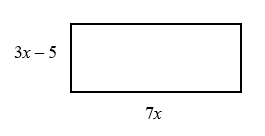
**a** 3(*y*2 – 8) – 4(*y*2 – 5) **b** 2*x*(*x* + 5) + 3*x*(*x* – 7)

**c** 4*p*(2*p* – 1) – 3*p*(5*p* – 2) **d** 3*b*(4*b* – 3) – *b*(6*b* – 9)

**5** Expand (2*y* – 8)

**6** Expand and simplify.

**a** 13 – 2(*m* + 7) **b** 5*p*(*p*2 + 6*p*) – 9*p*(2*p* – 3)

**7** The diagram shows a rectangle.

Write down an expression, in terms of *x*, for the area of the rectangle.

Show that the area of the rectangle can be written as 21*x*2– 35*x*

**8** Expand and simplify.

**a** (*x* + 4)(*x* + 5) **b** (*x* + 7)(*x* + 3)

**c** (*x* + 7)(*x* – 2) **d** (*x* + 5)(*x* – 5)

**e** (2*x* + 3)(*x* – 1) **f** (3*x* – 2)(2*x* + 1)

**g** (5*x* – 3)(2*x* – 5) **h** (3*x* – 2)(7 + 4*x*)

**i** (3*x* + 4*y*)(5*y* + 6*x*) **j** (*x* + 5)2

**k** (2*x* − 7)2 **l** (4*x* − 3*y*)2

Extend

**9** Expand and simplify (*x* + 3)² + (*x* − 4)²

**10** Expand and simplify.

**a**  **b** 

**2. Surds and rationalising the denominator**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* A surd is the square root of a number that is not a square number, for example  etc.
* Surds can be used to give the exact value for an answer.
* 
* 
* To rationalise the denominator means to remove the surd from the denominator of a fraction.
* To rationalise you multiply the numerator and denominator by the surd 
* To rationalise  you multiply the numerator and denominator by 

**Example 1** Simplify 

|  |  |
| --- | --- |
|  | **1** Choose two numbers that are factors of 50. One of the factors must be a square number  **2** Use the rule  **3** Use |

**Example 2** Simplify 

|  |  |
| --- | --- |
|  | **1** Simplify  and . Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number  **2** Use the rule  **3** Use  and  **4** Collect like terms |

**Example 3** Simplify 

|  |  |
| --- | --- |
| =  = 7 – 2  = 5 | **1** Expand the brackets. A common mistake here is to write  **2** Collect like terms: |

**Example 4** Rationalise 

|  |  |
| --- | --- |
| =  =  = | **1** Multiply the numerator and denominator by  **2** Use |

**Example 5** Rationalise and simplify 

|  |  |
| --- | --- |
| =  =  =  = | **1** Multiply the numerator and denominator by  **2** Simplify  in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number  **3** Use the rule  **4** Use  **5** Simplify the fraction:  simplifies to |

**Example 6** Rationalise and simplify 

|  |  |
| --- | --- |
| = =  =  =  =  = | **1** Multiply the numerator and denominator by  **2** Expand the brackets  **3** Simplify the fraction  **4** Divide the numerator by −1  Remember to change the sign of all terms when dividing by −1 |

Practice

**1** Simplify.

**Hint**

One of the two numbers you choose at the start must be a square number.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**2** Simplify.

**Watch out!**

Check you have chosen the highest square number at the start.

**a**  **b** 

**c**  **d** 

**e  f** 

**3** Expand and simplify.

**a**  **b** 

**c**  **d** 

**4** Rationalise and simplify, if possible.

**a**  **b**  **c**  **d** 

**e**  **f**  **g**  **h** 

**5** Rationalise and simplify.

**a**  **b**  **c** 

# **Extend**

**6** Expand and simplify 

**7** Rationalise and simplify, if possible.

**a**  **b** 

1. **Rules of indices**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* *am* × *an* = *am* + *n*
* 
* (*am*)*n* = *amn*
* *a*0 = 1
*  i.e. the *n*th root of *a*
* 
* 
* The square root of a number produces two solutions, e.g. .

Examples

**Example 1** Evaluate 100

|  |  |
| --- | --- |
| 100 = 1 | Any value raised to the power of zero is equal to 1 |

**Example 2** Evaluate 

|  |  |
| --- | --- |
| = 3 | Use the rule |

**Example 3** Evaluate 

|  |  |
| --- | --- |
| =  = 9 | **1** Use the rule  **2** Use |

**Example 4** Evaluate 

|  |  |
| --- | --- |
|  | **1** Use the rule  **2** Use |

**Example 5** Simplify 

|  |  |
| --- | --- |
| = 3*x*3 | 6 ÷ 2 = 3 and use the rule  to give |

**Example 6** Simplify 

|  |  |
| --- | --- |
| = *x*8 − 4 = *x*4 | **1** Use the rule  **2** Use the rule |

**Example 7** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | Use the rule , note that the fraction  remains unchanged |

**Example 8** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | **1** Use the rule  **2** Use the rule |

Practice

**1** Evaluate.

**a** 140 **b** 30 **c** 50 **d** *x*0

**2** Evaluate.

**a**  **b**  **c**  **d** 

**3** Evaluate.

**a**  **b**  **c**  **d** 

**4** Evaluate.

**a** 5–2 **b** 4–3 **c** 2–5 **d** 6–2

**5** Simplify.

**a**  **b**  **c**  **d** 

**e**  **f**  **g**  **h** 

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule *a*0 = 1.

**6** Evaluate.

**a**  **b**  **c** 

**d**  **e**  **f** 

**7** Write the following as a single power of *x*.

**a**  **b**  **c** 

**d**  **e**  **f** 

**8** Write the following without negative or fractional powers.

**a**  **b** *x*0 **c** 

**d**  **e**  **f** 

**9** Write the following in the form *axn*.

**a**  **b**  **c** 

**d**  **e**  **f** 3

# **Extend**

**10** Write as sums of powers of *x*.

**a**  **b**  **c** 

**4.** **Factorising expressions**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Factorising an expression is the opposite of expanding the brackets.
* A quadratic expression is in the form *ax*2 + *bx* + *c*, where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose product is *ac*.
* An expression in the form *x*2 – *y*2 is called the difference of two squares. It factorises to (*x* – *y*)(*x* + *y*).

Examples

**Example 1** Factorise 15*x*2*y*3 + 9*x*4*y*

|  |  |
| --- | --- |
| 15*x*2*y*3 + 9*x*4*y* = 3*x*2*y*(5*y*2 + 3*x*2) | The highest common factor is 3*x*2*y*. So take 3*x*2*y* outside the brackets and then divide each term by 3*x*2*y* to find the terms in the brackets |

**Example 2** Factorise 4*x*2 – 25*y*2

|  |  |
| --- | --- |
| 4*x*2 – 25*y*2 = (2*x* + 5*y*)(2*x* − 5*y*) | This is the difference of two squares as the two terms can be written as (2*x*)2and (5*y*)2 |

**Example 3** Factorise *x*2 + 3*x* – 10

|  |  |
| --- | --- |
| *b* = 3, *ac* = −10  So *x*2 + 3*x* – 10 = *x*2 + 5*x* – 2*x* – 10  = *x*(*x* + 5) – 2(*x* + 5)  = (*x* + 5)(*x* – 2) | **1** Work out the two factors of *ac*= −10 which add to give *b*= 3  (5 and −2)  **2** Rewrite the *b* term (3*x*) using these two factors  **3** Factorise the first two terms and the last two terms  **4** (*x* + 5) is a factor of both terms |

**Example 4** Factorise 6*x*2 − 11*x* − 10

|  |  |
| --- | --- |
| *b* = −11, *ac* = −60  So  6*x*2 − 11*x* – 10 =6*x*2 − 15*x* + 4*x* – 10  = 3*x*(2*x* − 5) + 2(2*x* − 5)  = (2*x* – 5)(3*x* + 2) | **1** Work out the two factors of *ac*= −60 which add to give *b*= −11 (−15 and 4)  **2** Rewrite the *b* term (−11*x*) using these two factors  **3** Factorise the first two terms and the last two terms  **4** (2*x* − 5) is a factor of both terms |

**Example 5** Simplify 

|  |  |
| --- | --- |
| For the numerator:  *b* = −4, *ac* = −21  So  *x*2 − 4*x* – 21 = *x*2 − 7*x* + 3*x* – 21  = *x*(*x* − 7) + 3(*x* − 7)  = (*x* – 7)(*x* + 3)  For the denominator:  *b* = 9, *ac* = 18  So  2*x*2 + 9*x* + 9 = 2*x*2 + 6*x* + 3*x* + 9  = 2*x*(*x* + 3) + 3(*x* + 3)  = (*x* + 3)(2*x* + 3)  So    = | **1** Factorise the numerator and the denominator  **2** Work out the two factors of *ac*= −21 which add to give *b*= −4 (−7 and 3)  **3** Rewrite the *b* term (−4*x*) using these two factors  **4** Factorise the first two terms and the last two terms  **5** (*x* − 7) is a factor of both terms  **6** Work out the two factors of  *ac*= 18 which add to give *b*= 9  (6 and 3)  **7** Rewrite the *b* term (9*x*) using these two factors  **8** Factorise the first two terms and the last two terms  **9** (*x* + 3) is a factor of both terms  **10** (*x* + 3) is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1 |

Practice

**1** Factorise.

**Hint**

Take the highest common factor outside the bracket.

**a** 6*x*4*y*3 – 10*x*3*y*4 **b** 21*a*3*b*5 + 35*a*5*b*2

**c** 25*x*2*y*2 – 10*x*3*y*2 + 15*x*2*y*3

**2** Factorise

**a** *x*2 + 7*x* + 12 **b** *x*2 + 5*x* – 14

**c** *x*2 – 11*x* + 30 **d** *x*2 – 5*x* – 24

**e** *x*2 – 7*x* – 18 **f** *x*2 + *x* –20

**g** *x*2 – 3*x* – 40 **h** *x*2 + 3*x* – 28

**3** Factorise

**a** 36*x*2 – 49*y*2 **b** 4*x*2 – 81*y*2

**c** 18*a*2 – 200*b*2*c*2

**4** Factorise

**a** 2*x*2 + *x* –3 **b** 6*x*2 + 17*x* + 5

**c** 2*x*2 + 7*x* + 3 **d** 9*x*2 – 15*x* + 4

**e** 10*x*2 + 21*x* + 9 **f** 12*x*2 – 38*x* + 20

**5** Simplify the algebraic fractions.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**6** Simplify

**a**  **b** 

**c**  **d** 

# **Extend**

**7** Simplify 

**8** Simplify 

**5. Completing the square**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Completing the square for a quadratic rearranges *ax*2 + *bx* + *c* into the form *p*(*x* + *q*)2 + *r*
* If *a* ≠ 1, then factorise using *a* as a common factor.

Examples

**Example 1** Complete the square for the quadratic expression *x*2 + 6*x* − 2

|  |  |
| --- | --- |
| *x*2 + 6*x* − 2  = (*x* + 3)2 − 9 − 2  = (*x* + 3)2 − 11 | **1** Write *x*2 + *bx* + *c* in the form  **2** Simplify |

**Example 2** Write 2*x*2 − 5*x* + 1 in the form *p*(*x* + *q*)2 + *r*

|  |  |
| --- | --- |
| 2*x*2 − 5*x* + 1  =  =  =  = | **1** Before completing the square write *ax*2 + *bx* + *c* in the form  **2** Now complete the square by writing  in the form  **3** Expand the square brackets – don’t forget to multiply by the factor of 2  **4** Simplify |

Practice

**1** Write the following quadratic expressions in the form (*x* + *p*)2 + *q*

**a** *x*2 + 4*x* + 3 **b** *x*2 – 10*x* – 3

**c** *x*2 – 8*x* **d** *x*2 + 6*x*

**e** *x*2 – 2*x* + 7 **f** *x*2 + 3*x* – 2

**2** Write the following quadratic expressions in the form *p*(*x* + *q*)2 + *r*

**a** 2*x*2 – 8*x* – 16 **b** 4*x*2 – 8*x* – 16

**c** 3*x*2 + 12*x* – 9 **d** 2*x*2 + 6*x* – 8

**3** Complete the square.

**a** 2*x*2 + 3*x* + 6 **b** 3*x*2 – 2*x*

**c** 5*x*2 + 3*x* **d** 3*x*2 + 5*x* + 3

Extend

**4** Write (25*x*2 + 30*x* + 12) in the form (*ax* + *b*)2 + *c*.

**6a. Solving quadratic equations by factorisation**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* A quadratic equation is an equation in the form *ax*2 + *bx* + *c* = 0 where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose products is *ac*.
* When the product of two numbers is 0, then at least one of the numbers must be 0.
* If a quadratic can be solved it will have two solutions (these may be equal).

Examples

**Example 1** Solve 5*x*2 = 15*x*

|  |  |
| --- | --- |
| 5*x*2 = 15*x*  5*x*2 − 15*x* = 0  5*x*(*x* − 3) = 0  So 5*x* = 0 or (*x* − 3) = 0  Therefore *x* = 0 or *x* = 3 | **1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero.  Do not divide both sides by *x* as this would lose the solution *x* = 0.  **2** Factorise the quadratic equation.  5*x* is a common factor.  **3** When two values multiply to make zero, at least one of the values must be zero.  **4** Solve these two equations. |

**Example 2** Solve *x*2 + 7*x* + 12 = 0

|  |  |
| --- | --- |
| *x*2 + 7*x* + 12 = 0  *b* = 7, *ac* = 12  *x*2 + 4*x* + 3*x* + 12 = 0  *x*(*x* + 4) + 3(*x* + 4) = 0  (*x* + 4)(*x* + 3) = 0  So (*x* + 4)= 0 or (*x* + 3) = 0  Therefore *x* = −4 or *x* = −3 | **1** Factorise the quadratic equation. Work out the two factors of *ac* = 12 which add to give you *b* = 7.  (4 and 3)  **2** Rewrite the *b* term (7*x*) using these two factors.  **3** Factorise the first two terms and the last two terms.  **4** (*x* + 4) is a factor of both terms.  **5** When two values multiply to make zero, at least one of the values must be zero.  **6** Solve these two equations. |

**Example 3** Solve 9*x*2 − 16 = 0

|  |  |
| --- | --- |
| 9*x*2 − 16 = 0  (3*x* + 4)(3*x* – 4) = 0  So (3*x* + 4) = 0 or (3*x* – 4) = 0  or | **1** Factorise the quadratic equation. This is the difference of two squares as the two terms are (3*x*)2 and (4)2.  **2** When two values multiply to make zero, at least one of the values must be zero.  **3** Solve these two equations. |

**Example 4** Solve 2*x*2 − 5*x* − 12 = 0

|  |  |
| --- | --- |
| *b* = −5, *ac* = −24  So 2*x*2 − 8*x* + 3*x* – 12 = 0  2*x*(*x* − 4) + 3(*x* − 4) = 0  (*x* – 4)(2*x* + 3) = 0  So (*x* – 4) = 0 or (2*x* +3) = 0  or | **1** Factorise the quadratic equation. Work out the two factors of *ac* = −24 which add to give you *b* = −5.  (−8 and 3)  **2** Rewrite the *b* term (−5*x*) using these two factors.  **3** Factorise the first two terms and the last two terms.  **4** (*x* − 4) is a factor of both terms.  **5** When two values multiply to make zero, at least one of the values must be zero.  **6** Solve these two equations. |

Practice

**1** Solve

**a** 6*x*2 + 4*x* = 0 **b** 28*x*2 – 21*x* = 0

**c** *x*2 + 7*x* + 10 = 0 **d** *x*2 – 5*x* + 6 = 0

**e** *x*2 – 3*x* – 4 = 0 **f** *x*2 + 3*x* – 10 = 0

**g** *x*2 – 10*x* + 24 = 0 **h** *x*2 – 36 = 0

**i** *x*2 + 3*x* – 28 = 0 **j** *x*2 – 6*x* + 9 = 0

**k** 2*x*2 – 7*x* – 4 = 0 **l** 3*x*2 – 13*x* – 10 = 0

**2** Solve

**Hint**

Get all terms onto one side of the equation.

**a** *x*2 – 3*x* = 10 **b** *x*2 – 3 = 2*x*

**c** *x*2 + 5*x* = 24 **d** *x*2 – 42 = *x*

**e** *x*(*x* + 2) = 2*x* + 25 **f** *x*2 – 30 = 3*x* – 2

**g** *x*(3*x* + 1) = *x*2 + 15 **h** 3*x*(*x* – 1) = 2(*x* + 1)

**6b. Solving quadratic equations by completing the square**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Completing the square lets you write a quadratic equation in the form *p*(*x* + *q*)2 + *r* = 0*.*

Examples

**Example 5** Solve *x*2 + 6*x* + 4 = 0. Give your solutions in surd form.

|  |  |
| --- | --- |
| *x*2 + 6*x* + 4 = 0  (*x* + 3)2 − 9 + 4 = 0  (*x* + 3)2 − 5 = 0  (*x* + 3)2 = 5  *x* + 3 =  *x* =  So *x* =  or *x* = | **1** Write *x*2 + *bx* + *c* = 0 in the form  **2** Simplify.  **3** Rearrange the equation to work out *x*. First, add 5 to both sides.  **4** Square root both sides.  Remember that the square root of a value gives two answers.  **5** Subtract 3 from both sides to solve the equation.  **6** Write down both solutions. |

**Example 6** Solve 2*x*2 − 7*x* + 4 = 0. Give your solutions in surd form.

|  |  |
| --- | --- |
| 2*x*2 − 7*x* + 4 = 0  = 0  = 0  = 0  = 0          So  or | **1** Before completing the square write *ax*2 + *bx* + *c* in the form  **2** Now complete the square by writing  in the form  **3** Expand the square brackets.  **4** Simplify.  **5** Rearrange the equation to work out *x*. First, add  to both sides.  **6** Divide both sides by 2.  **7** Square root both sides. Remember that the square root of a value gives two answers.  **8** Add  to both sides.  **9** Write down both the solutions. |

Practice

**3** Solve by completing the square.

**a** *x*2 – 4*x* – 3 = 0 **b** *x*2 – 10*x* + 4 = 0

**c** *x*2 + 8*x* – 5 = 0 **d** *x*2 – 2*x* – 6 = 0

**e** 2*x*2 + 8*x* – 5 = 0 **f** 5*x*2 + 3*x* – 4 = 0

**4** Solve by completing the square.

**Hint**

Get all terms onto one side of the equation.

**a** (*x* – 4)(*x* + 2) = 5

**b** 2*x*2 + 6*x* – 7 = 0

**c** *x*2 – 5*x* + 3 = 0

**6c. Solving quadratic equations by using the formula**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Any quadratic equation of the form *ax*2 + *bx* + *c* = 0 can be solved using the formula 
* If *b*2 – 4*ac* is negative then the quadratic equation does not have any real solutions.
* It is useful to write down the formula before substituting the values for *a*, *b* and *c*.

Examples

**Example 7** Solve *x*2 + 6*x* + 4 = 0. Give your solutions in surd form.

|  |  |
| --- | --- |
| *a* = 1, *b* = 6, *c* = 4            So  or | **1** Identify *a*, *b* and *c* and write down the formula.  Remember that  is all over 2*a*, not just part of it.  **2** Substitute *a* = 1, *b* = 6, *c* = 4 into the formula.  **3** Simplify. The denominator is 2, but this is only because *a* = 1. The denominator will not always be 2.  **4** Simplify .  **5** Simplify by dividing numerator and denominator by 2.  **6** Write down both the solutions. |

**Example 8** Solve 3*x*2 − 7*x* − 2 = 0. Give your solutions in surd form.

|  |  |
| --- | --- |
| *a* = 3, *b* = −7, *c* = −2        So  or | **1** Identify *a*, *b* and *c*, making sure you get the signs right and write down the formula.  Remember that  is all over 2*a*, not just part of it.  **2** Substitute *a* = 3, *b* = −7, *c* = −2 into the formula.  **3** Simplify. The denominator is 6 when *a* = 3. A common mistake is to always write a denominator of 2.  **4** Write down both the solutions. |

Practice

**5** Solve, giving your solutions in surd form.

**a** 3*x*2 + 6*x* + 2 = 0 **b** 2*x*2 – 4*x* – 7 = 0

**6** Solve the equation *x*2 – 7*x* + 2 = 0

Give your solutions in the form , where *a*, *b* and *c* are integers.

**7** Solve 10*x*2 + 3*x* + 3 = 5

**Hint**

Get all terms onto one side of the equation.

Give your solution in surd form.

Extend

**8** Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

**a** 4*x*(*x* – 1) = 3*x* – 2

**b** 10 = (*x* + 1)2

**c** *x*(3*x* – 1) = 10

**7. Sketching quadratic graphs**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* The graph of the quadratic function   
  *y* = *ax*2 + *bx* + *c*, where *a* ≠ 0, is a curve   
  called a parabola.

for *a* < 0

for *a* > 0

* Parabolas have a line of symmetry and   
  a shape as shown.
* To sketch the graph of a function, find the points where the graph intersects the axes.
* To find where the curve intersects the *y*-axis substitute *x* = 0 into the function.
* To find where the curve intersects the *x*-axis substitute *y* = 0 into the function.
* At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
* To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

**Example 1** Sketch the graph of *y* = *x*2.

|  |  |
| --- | --- |
|  | The graph of *y* = *x*2 is a parabola.  When *x* = 0, *y* = 0.  *a* = 1 which is greater than zero, so the graph has the shape: |

**Example 2** Sketch the graph of *y* = *x*2 − *x* − 6.

|  |  |
| --- | --- |
| When *x* = 0, *y* = 02 − 0 − 6 = −6  So the graph intersects the *y*-axis at  (0, −6)  When *y* = 0, *x*2 − *x* − 6 = 0  (*x* + 2)(*x* − 3) = 0  *x* = −2 or *x* = 3  So,  the graph intersects the *x*-axis at (−2, 0) and (3, 0)  *x*2 − *x* − 6 = =  When ,  and , so the turning point is at the point | **1** Find where the graph intersects the *y*-axis by substituting *x* = 0.  **2** Find where the graph intersects the *x*-axis by substituting *y* = 0.  **3** Solve the equation by factorising.  **4** Solve (*x* + 2) = 0 and (*x* − 3) = 0.  **5** *a* = 1 which is greater than zero, so the graph has the shape:  **6** To find the turning point, complete the square.  **7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero. |

Practice

**1** Sketch the graph of *y* = −*x*2.

**2** Sketch each graph, labelling where the curve crosses the axes.

**a** *y* = (*x* + 2)(*x* − 1) **b** *y* = *x*(*x* − 3) **c** *y* = (*x* + 1)(*x* + 5)

**3** Sketch each graph, labelling where the curve crosses the axes.

**a** *y* = *x*2 − *x* − 6 **b** *y* = *x*2 − 5*x* + 4 **c** *y* = *x*2 – 4

**d** *y* = *x*2 + 4*x* **e** *y* = 9 − *x*2 **f** *y* = *x*2 + 2*x* − 3

**4** Sketch the graph of *y* = 2*x*2 + 5*x* − 3, labelling where the curve crosses the axes.

Extend

**5** Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

**a** *y* = *x*2 − 5*x* + 6 **b** *y* = −*x*2 + 7*x* − 12 **c** *y* = −*x*2 + 4*x*

**6** Sketch the graph of *y* = *x*2 + 2*x* + 1. Label where the curve crosses the axes and write down the equation of the line of symmetry.

**8a. Solving linear simultaneous equations using the elimination method**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* Two equations are simultaneous when they are both true at the same time.
* Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
* Make sure that the coefficient of one of the unknowns is the same in both equations.
* Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

**Example 1** Solve the simultaneous equations 3*x* + *y* = 5 and *x* + *y* = 1

|  |  |
| --- | --- |
| 3*x* + *y* = 5  *– x* + *y* = 1  2*x* = 4  So *x* = 2  Using *x* + *y* = 1  2 + *y* = 1  So *y* = −1  Check:  equation 1: 3 × 2 + (−1) = 5 YES  equation 2: 2 + (−1) = 1 YES | **1** Subtract the second equation from the first equation to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 2 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 2** Solve *x* + 2*y* = 13 and 5*x* − 2*y* = 5 simultaneously.

|  |  |
| --- | --- |
| *x* + 2*y* = 13  + 5*x* − 2*y* = 5     6*x* = 18  So *x* = 3  Using *x* + 2*y* = 13  3 + 2*y* = 13  So *y* = 5  Check:  equation 1: 3 + 2 × 5 = 13 YES  equation 2: 5 × 3 − 2 × 5 = 5 YES | **1** Add the two equations together to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 3 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 3** Solve 2*x* + 3*y* = 2 and 5*x* + 4*y* = 12 simultaneously.

|  |  |
| --- | --- |
| (2*x* + 3*y* = 2) × 4  8*x* + 12*y* = 8  (5*x* + 4*y* = 12) × 3 15*x* + 12*y* = 36  7*x* = 28  So *x* = 4  Using 2*x* + 3*y*  = 2  2 × 4 + 3*y* = 2  So *y* = −2  Check:  equation 1: 2 × 4 + 3 × (−2) = 2 YES  equation 2: 5 × 4 + 4 × (−2) = 12 YES | **1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of *y* the same for both equations. Then subtract the first equation from the second equation to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 4 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**1** 4*x* + *y* = 8 **2** 3*x* + *y* = 7

*x* + *y* = 5 3*x* + 2*y* = 5

**3** 4*x* + *y* = 3 **4** 3*x* + 4*y* = 7

3*x* – *y* = 11 *x* – 4*y* = 5

**5** 2*x* + *y* = 11 **6** 2*x* + 3*y* = 11

*x* – 3*y* = 9 3*x* + 2*y* = 4

**8b. Solving linear simultaneous equations using the substitution method**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

**Textbook:**Pure Year 1, 3.1 Linear simultaneous equations

Key points

* The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

**Example 4** Solve the simultaneous equations *y* = 2*x* + 1 and 5*x* + 3*y* = 14

|  |  |
| --- | --- |
| 5*x* + 3(2*x* + 1) = 14  5*x* + 6*x* + 3 = 14  11*x* + 3 = 14  11*x* = 11  So *x* = 1  Using *y* = 2*x* + 1  *y* = 2 × 1 + 1  So *y* = 3  Check:  equation 1: 3 = 2 × 1 + 1 YES  equation 2: 5 × 1 + 3 × 3 = 14 YES | **1** Substitute 2*x* + 1 for *y* into the second equation.  **2** Expand the brackets and simplify.  **3** Work out the value of *x*.  **4** To find the value of *y*, substitute *x*= 1 into one of the original equations.  **5** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 5** Solve 2*x* − *y* = 16 and 4*x* + 3*y* = −3 simultaneously.

|  |  |
| --- | --- |
| *y* = 2*x* − 16  4*x* + 3(2*x* − 16) = −3  4*x* + 6*x* − 48 = −3  10*x* − 48 = −3  10*x* = 45  So *x* =  Using *y* = 2*x* − 16  *y* = 2 ×  − 16  So *y* = −7  Check:  equation 1: 2 ×  – (–7) = 16 YES  equation 2: 4 ×  + 3 × (−7) = −3 YES | **1** Rearrange the first equation.  **2** Substitute 2*x* − 16 for *y* into the second equation.  **3** Expand the brackets and simplify.  **4** Work out the value of *x*.  **5** To find the value of *y*, substitute *x*=  into one of the original equations.  **6** Substitute the values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**7** *y* = *x* –4 **8** *y* = 2*x* – 3

2*x* + 5*y* = 43 5*x* – 3*y* = 11

**9** 2*y* = 4*x* + 5 **10** 2*x* = *y* – 2

9*x* + 5*y* = 22 8*x* – 5*y* = –11

**11** 3*x* + 4*y* = 8 **12** 3*y* = 4*x* – 7

2*x* – *y* = –13 2*y* = 3*x* – 4

**13** 3*x* = *y* – 1 **14** 3*x* + 2*y* + 1 = 0

2*y* – 2*x* = 3 4*y* = 8 – *x*

Extend

**15** Solve the simultaneous equations 3*x* + 5*y* − 20 = 0 and .

**9. Solving linear and quadratic simultaneous equations**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* Make one of the unknowns the subject of the linear equation (rearranging where necessary).
* Use the linear equation to substitute into the quadratic equation.
* There are usually two pairs of solutions.

Examples

**Example 1** Solve the simultaneous equations *y* = *x* + 1 and *x*2 + *y*2 = 13

|  |  |
| --- | --- |
| *x*2 + (*x* + 1)2 = 13  *x*2 + *x*2 + *x* + *x* + 1 = 13  2*x*2 + 2*x* + 1 = 13  2*x*2 + 2*x* − 12 = 0  (2*x* − 4)(*x* + 3) = 0  So *x* = 2 or *x* = −3  Using *y* = *x* + 1  When *x* = 2, *y* = 2 + 1 = 3  When *x* = −3, *y* = −3 + 1 = −2  So the solutions are  *x* = 2, *y* = 3 and *x* = −3, *y* = −2  Check:  equation 1: 3 = 2 + 1 YES  and −2 = −3 + 1 YES  equation 2: 22 + 32 = 13 YES  and (−3)2 + (−2)2 = 13 YES | **1** Substitute *x* + 1 for *y* into the second equation.  **2** Expand the brackets and simplify.  **3** Factorise the quadratic equation.  **4** Work out the values of *x*.  **5** To find the value of *y*, substitute both values of *x* into one of the original equations.  **6** Substitute both pairs of values of *x* and *y* into both equations to check your answers. |

**Example 2** Solve 2*x* + 3*y* = 5 and 2*y*2 + *xy* = 12 simultaneously.

|  |  |
| --- | --- |
| (*y* + 8)(*y* − 3) = 0  So *y* = −8 or *y* = 3  Using 2*x* + 3*y* = 5  When *y* = −8, 2*x* + 3 × (−8) = 5, *x* = 14.5  When *y* = 3, 2*x* + 3 × 3 = 5, *x* = −2  So the solutions are  *x* = 14.5, *y* = −8 and *x* = −2, *y* = 3  Check:  equation 1: 2 × 14.5 + 3 × (−8) = 5 YES  and 2 × (−2) + 3 × 3 = 5 YES  equation 2: 2×(−8)2 + 14.5×(−8) = 12 YES  and 2 × (3)2 + (−2) × 3 = 12 YES | **1** Rearrange the first equation.  **2** Substitute  for *x* into the second equation. Notice how it is easier to substitute for *x* than for *y*.  **3** Expand the brackets and simplify.  **4** Factorise the quadratic equation.  **5** Work out the values of *y*.  **6** To find the value of *x*, substitute both values of *y* into one of the original equations.  **7** Substitute both pairs of values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**1** *y* = 2*x* + 1 **2** *y* = 6 – *x* **3** *y* = *x* – 3

*x*2 + *y*2 = 10 *x*2 + *y*2 = 20 *x*2 + *y*2 = 5

**4** *y* = 9 − 2*x* **5** *y* = 3*x* – 5 **6** *y* = *x* − 5

*x*2 + *y*2 = 17 *y* = *x*2 − 2*x* + 1 *y* = *x*2 − 5*x* − 12

**7** *y* = *x* + 5 **8** *y* = 2*x* – 1 **9** *y* = 2*x*

*x*2 + *y*2 = 25 *x*2 + *xy* = 24 *y*2 – *xy* = 8

**10** 2*x* + *y* = 11

*xy* = 15

Extend

**11** *x* – *y* = 1 **12** *y* – *x* = 2

*x*2 + *y*2 = 3 *x*2 + *xy* = 3**10.** **Solving simultaneous equations graphically**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Examples

**Example 1** Solve the simultaneous equations *y* = 5*x* + 2 and *x* + *y* = 5 graphically.

|  |  |
| --- | --- |
| *y* = 5 – *x*  *y* = 5 – *x* has gradient –1 and *y*-intercept 5.  *y* = 5*x* + 2 has gradient 5 and *y*-intercept 2.    Lines intersect at  *x* = 0.5, *y* = 4.5  Check:  First equation *y* = 5*x* + 2:  4.5 = 5 × 0.5 + 2 YES  Second equation *x* + *y* = 5:  0.5 + 4.5 = 5 YES | **1** Rearrange the equation *x* + *y* = 5 to make *y* the subject.  **2** Plot both graphs on the same grid using the gradients and *y*-intercepts.  **3** The solutions of the simultaneous equations are the point of intersection.  **4** Check your solutions by substituting the values into both equations. |

**Example 2** Solve the simultaneous equations *y* = *x* − 4 and *y* = *x*2 − 4*x* + 2 graphically.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | ***x*** | 0 | 1 | 2 | 3 | 4 | | ***y*** | 2 | –1 | –2 | –1 | 2 |     The line and curve intersect at  *x* = 3, *y* = −1 and *x* = 2, *y* = −2  Check:  First equation *y* = *x* − 4:  −1 = 3 − 4 YES  −2 = 2 − 4 YES  Second equation *y* = *x*2 − 4*x* + 2:  −1 = 32 − 4 × 3 + 2 YES  −2 = 22 − 4 × 2 + 2 YES | **1** Construct a table of values and calculate the points for the quadratic equation.  **2** Plot the graph.  **3** Plot the linear graph on the same grid using the gradient and  *y*-intercept. *y* = *x* – 4 has gradient 1 and  *y*-intercept –4.  **4** The solutions of the simultaneous equations are the points of intersection.  **5** Check your solutions by substituting the values into both equations. |

Practice

**1** Solve these pairs of simultaneous equations graphically.

**a** *y* = 3*x* − 1 and *y* = *x* + 3

**b** *y* = *x* − 5 and *y* = 7 − 5*x*

**c** *y* = 3*x* + 4 and *y* = 2 − *x*

**2** Solve these pairs of simultaneous equations graphically.

**Hint**

Rearrange the equation to make *y* the subject.

**a** *x* + *y* = 0 and *y* = 2*x* + 6

**b** 4*x* + 2*y* = 3 and *y* = 3*x* − 1

**c** 2*x* + *y* + 4 = 0 and 2*y* = 3*x* − 1

**3** Solve these pairs of simultaneous equations graphically.

**a** *y* = *x* − 1 and *y* = *x*2 − 4*x* + 3

**b** *y* = 1 − 3*x* and *y* = *x*2 − 3*x* − 3

**c** *y* = 3 − *x* and *y* = *x*2 + 2*x* + 5

**4** Solve the simultaneous equations *x* + *y* = 1 and *x*2 + *y*2 = 25 graphically.

Extend

**5** **a** Solve the simultaneous equations 2*x* + *y* = 3 and *x*2 + *y* = 4

**i** graphically

**ii** algebraically to 2 decimal places.

**b** Which method gives the more accurate solutions? Explain your answer.

**11. Linear inequalities**

**A LEVEL LINKS**

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

* Solving linear inequalities uses similar methods to those for solving linear equations.
* When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

**Example 1** Solve −8 ≤ 4*x* < 16

|  |  |
| --- | --- |
| −8 ≤ 4*x* < 16  −2 ≤ *x*  < 4 | Divide all three terms by 4. |

**Example 2** Solve 4 ≤ 5*x* < 10

|  |  |
| --- | --- |
| 4 ≤ 5*x* < 10  ≤ *x* < 2 | Divide all three terms by 5. |

**Example 3** Solve 2*x* − 5 < 7

|  |  |
| --- | --- |
| 2*x* − 5 < 7  2*x* < 12  *x* < 6 | **1** Add 5 to both sides.  **2** Divide both sides by 2. |

**Example 4** Solve 2 − 5*x* ≥ −8

|  |  |
| --- | --- |
| 2 − 5*x* ≥ −8  −5*x* ≥ −10  *x* ≤ 2 | **1** Subtract 2 from both sides.  **2** Divide both sides by −5.  Remember to reverse the inequality when dividing by a negative number. |

**Example 5** Solve 4(*x* − 2) > 3(9 − *x*)

|  |  |
| --- | --- |
| 4(*x* − 2) > 3(9 − *x*)  4*x* − 8 > 27 − 3*x*  7*x* − 8 > 27  7*x* > 35  *x* > 5 | **1** Expand the brackets.  **2** Add 3*x* to both sides.  **3** Add 8 to both sides.  **4** Divide both sides by 7. |

Practice

**1** Solve these inequalities.

**a** 4*x* > 16 **b** 5*x* – 7 ≤ 3 **c** 1 ≥ 3*x* + 4

**d** 5 – 2*x* < 12 **e**  **f** 8 < 3 – 

**2** Solve these inequalities.

**a**  **b** 10 ≥ 2*x* + 3 **c** 7 – 3*x* > –5

**3** Solve

**a** 2 – 4*x* ≥ 18 **b** 3 ≤ 7*x* + 10 < 45 **c** 6 – 2*x* ≥ 4

**d** 4*x* + 17 < 2 – *x* **e** 4 – 5*x* < –3*x* **f** –4*x* ≥ 24

**4** Solve these inequalities.

**a** 3*t* + 1 < *t* + 6 **b** 2(3*n* – 1) ≥ *n* + 5

**5** Solve.

**a** 3(2 – *x*) > 2(4 – *x*) + 4 **b** 5(4 – *x*) > 3(5 – *x*) + 2

Extend

**6** Find the set of values of *x* for which 2*x* + 1 > 11 and 4*x* – 2 > 16 – 2*x*.

**12. Quadratic inequalities**

**A LEVEL LINKS**

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

* First replace the inequality sign by = and solve the quadratic equation.
* Sketch the graph of the quadratic function.
* Use the graph to find the values which satisfy the quadratic inequality.

Examples

**Example 1** Find the set of values of *x* which satisfy *x*2 + 5*x* + 6 > 0

|  |  |
| --- | --- |
| *x*2 + 5*x* + 6 = 0  (*x* + 3)(*x* + 2) = 0  *x* = −3 or *x* = −2    *x* < −3 or *x* > −2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of  *y* = (*x* + 3)(*x* + 2)  **3** Identify on the graph where  *x*2 + 5*x* + 6 > 0, i.e. where *y* > 0  **4** Write down the values which satisfy the inequality *x*2 + 5*x* + 6 > 0 |

**Example 2** Find the set of values of *x* which satisfy *x*2 − 5*x* ≤ 0

|  |  |
| --- | --- |
| *x*2 − 5*x* = 0  *x*(*x* − 5) = 0  *x* = 0 or *x* = 5    0 ≤ *x* ≤ 5 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = *x*(*x* − 5)  **3** Identify on the graph where  *x*2 − 5*x* ≤ 0, i.e. where *y* ≤ 0  **4** Write down the values which satisfy the inequality *x*2 − 5*x* ≤ 0 |

**Example 3** Find the set of values of *x* which satisfy −*x*2 − 3*x* + 10 ≥ 0

|  |  |
| --- | --- |
| −*x*2 − 3*x* + 10 = 0  (−*x* + 2)(*x* + 5) = 0  *x* = 2 or *x* = −5    −5 ≤ *x* ≤ 2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = (−*x* + 2)(*x* + 5) = 0  **3** Identify on the graph where −*x*2 − 3*x* + 10 ≥ 0, i.e. where *y* ≥ 0  **3** Write down the values which satisfy the inequality −*x*2 − 3*x* + 10 ≥ 0 |

Practice

**1** Find the set of values of *x* for which (*x* + 7)(*x* – 4) ≤ 0

**2** Find the set of values of *x* for which *x*2 – 4*x* – 12 ≥ 0

**3** Find the set of values of *x* for which 2*x*2 –7*x* + 3 < 0

**4** Find the set of values of *x* for which 4*x*2 + 4*x* – 3 > 0

**5** Find the set of values of *x* for which 12 + *x* – *x*2 ≥ 0

Extend

Find the set of values which satisfy the following inequalities.

**6** *x*2 + *x* ≤ 6

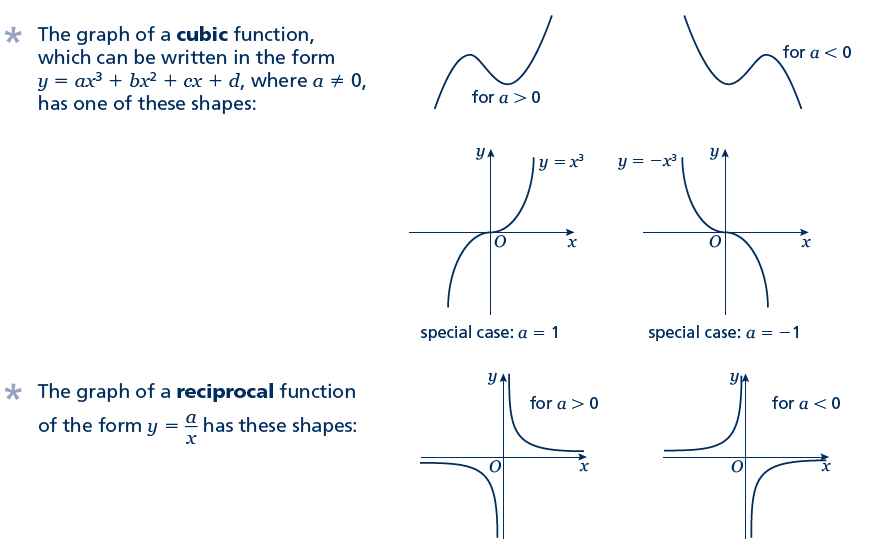
**7** *x*(2*x* – 9) < –10

**8** 6*x*2 ≥ 15 + *x*

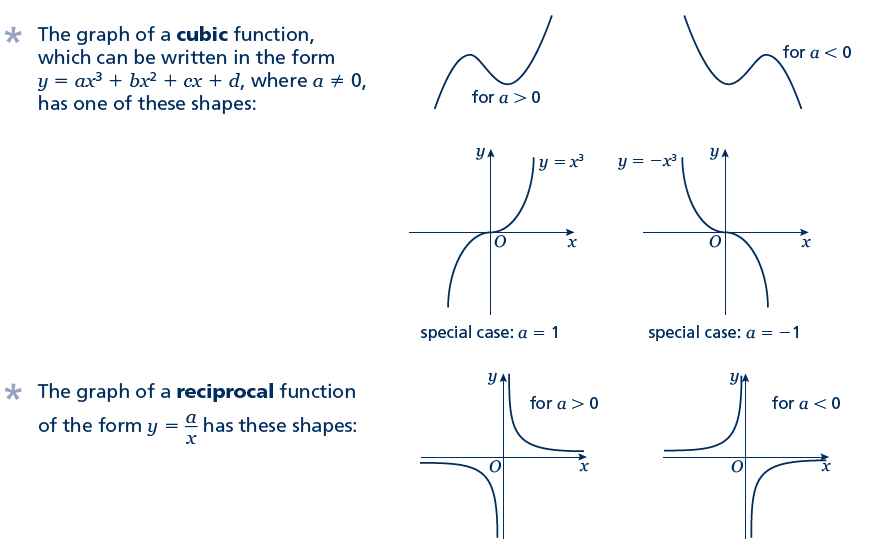
**13. Sketching cubic and reciprocal graphs**

**A LEVEL LINKS**

**Scheme of work:** 1e. Graphs – cubic, quartic and reciprocal

Key points

* The graph of a cubic function, which can be written in the form *y* = *ax*3 + *bx*2 + *cx* + *d*, where *a* ≠ 0, has one of the shapes shown here.



* The graph of a reciprocal function of the form  has one of the shapes shown here.
* To sketch the graph of a function, find the points where the graph intersects the axes.
* To find where the curve intersects the *y*-axis substitute *x* = 0 into the function.
* To find where the curve intersects the *x*-axis substitute *y* = 0 into the function.
* Where appropriate, mark and label the asymptotes on the graph.
* Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of  are the two axes (the lines *y* = 0 and *x* = 0).
* At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
* A double root is when two of the solutions are equal. For example (*x* – 3)2(*x* + 2) has a double root at *x* = 3.
* When there is a double root, this is one of the turning points of a cubic function.

Examples

**Example 1** Sketch the graph of *y* = (*x* − 3)(*x* − 1)(*x* + 2)

|  |  |
| --- | --- |
| To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape. | |
| When *x* = 0, *y* = (0 − 3)(0 − 1)(0 + 2)  = (−3) × (−1) × 2 = 6  The graph intersects the *y*-axis at (0, 6)  When *y* = 0, (*x* − 3)(*x* − 1)(*x* + 2) = 0  So *x* = 3, *x* = 1 or *x* = −2  The graph intersects the *x*-axis at   (−2, 0), (1, 0) and (3, 0) | **1** Find where the graph intersects the axes by substituting *x* = 0 and *y* = 0.  Make sure you get the coordinates the right way around, (*x*, *y*).  **2** Solve the equation by solving  *x* − 3 = 0, *x* − 1 = 0 and *x* + 2 = 0  **3** Sketch the graph.  *a* = 1 > 0 so the graph has the shape: |

**Example 2** Sketch the graph of *y* = (*x* + 2)2(*x* − 1)

|  |  |
| --- | --- |
| To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape. | |
| When *x* = 0, *y* = (0 + 2)2(0 − 1)  = 22 × (−1) = −4  The graph intersects the *y*-axis at (0, −4)  When *y* = 0, (*x* + 2)2(*x* − 1) = 0  So *x* = −2 or *x* =1  (−2, 0) is a turning point as *x* = −2 is a double root. The graph crosses the *x*-axis at (1, 0) | **1** Find where the graph intersects the axes by substituting *x* = 0 and *y* = 0.  **2** Solve the equation by solving  *x* + 2 = 0 and *x* − 1 = 0  **3** *a* = 1 > 0 so the graph has the shape: |

Practice

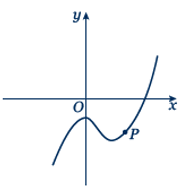
**1** Here are six equations.

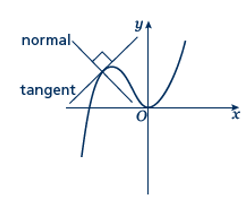
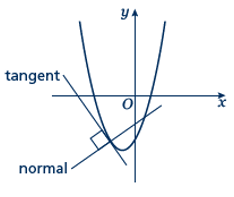
**Hint**

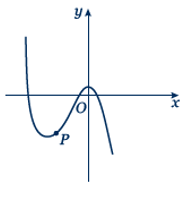
Find where each of the cubic equations cross the *y*-axis.

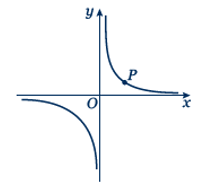
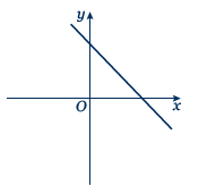
**A**  **B** *y* = *x*2 + 3*x* – 10 **C** *y* = *x*3 + 3*x*2

**D** *y* = 1 – 3*x*2 – *x*3 **E** *y* = *x*3 – 3*x*2 – 1 **F** *x* + *y* = 5

 Here are six graphs.

** i ii iii**

****

** iv v vi**

**a** Match each graph to its equation.

**b** Copy the graphs ii, iv and vi and draw the tangent and normal each at point *P*.

Sketch the following graphs

**2**  *y* = 2*x*3 **3** *y* = *x*(*x* – 2)(*x* + 2)

**4** *y* = (*x* + 1)(*x* + 4)(*x* – 3) **5** *y* = (*x* + 1)(*x* – 2)(1 – *x*)

**6** *y* = (*x* – 3)2(*x* + 1) **7** *y* = (*x* – 1)2(*x* – 2)

**8** *y* =  **9** *y* = 

**Hint:** Look at the shape of *y* =  in the second key point.

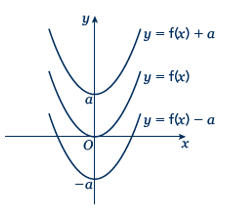
Extend

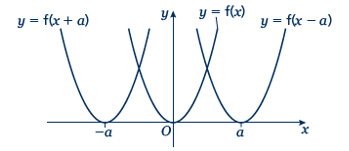
**10** Sketch the graph of  **11** Sketch the graph of 

**14a. Translating graphs**

**A LEVEL LINKS**

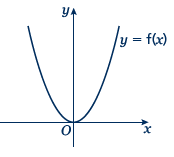
**Scheme of work:** 1f. Transformations – transforming graphs – f(*x*) notation

Key points

* The transformation *y* = f(*x*) ± *a* is a translation of *y* = f(*x*) parallel to the *y*-axis; it is a vertical translation.   
  As shown on the graph,
  + *y* = f(*x*) + *a* translates *y* = f(*x*) up
  + *y* = f(*x*) – *a* translates *y* = f(*x*) down.
* The transformation *y =* f(*x ± a*) is a translation of *y* = f(*x*) parallel to the *x*-axis; it is a horizontal translation.

As shown on the graph,

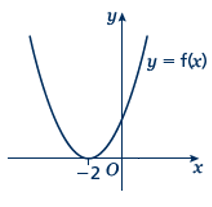
* + *y* = f(*x* + *a*) translates *y* = f(*x*) to the left
  + *y* = f(*x* – *a*) translates *y* = f(*x*) to the right.

Examples

**Example 1** The graph shows the function *y* = f(*x*).

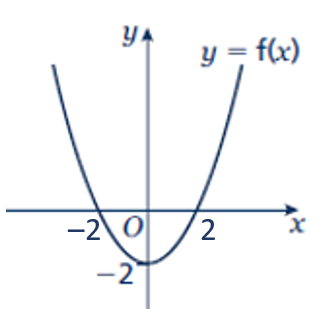
Sketch the graph of *y* = f(*x*) + 2.

|  |  |
| --- | --- |
|  | For the function *y* = f(*x*) + 2 translate the function *y* = f(*x*) 2 units up. |

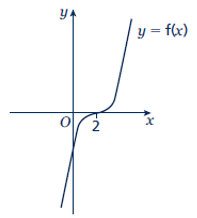
**Example 2** The graph shows the function *y* = f(*x*).

Sketch the graph of *y* = f(*x* − 3).

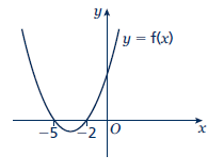
|  |  |
| --- | --- |
|  | For the function *y* = f(*x* − 3) translate the function *y* = f(*x*) 3 units right. |

Practice

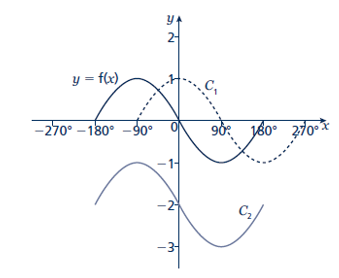
**1** The graph shows the function *y* = f(*x*).   
Copy the graph and on the same axes sketch and label the graphs of *y* = f(*x*) + 4 and *y* = f(*x* + 2).



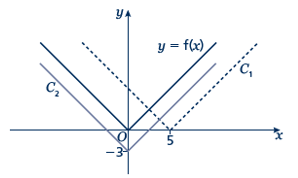
**2** The graph shows the function *y* = f(*x*).  
Copy the graph and on the same axes sketch and label the graphs of *y* = f(*x* + 3) and *y* = f(*x*) – 3.

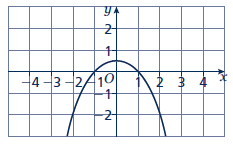


**3** The graph shows the function *y* = f(*x*).  
Copy the graph and on the same axes sketch the graph of *y* = f(*x* – 5).



**4** The graph shows the function *y* = f(*x*) and two transformations of *y* = f(*x*), labelled *C*1 and *C*2.  
Write down the equations of the translated curves *C*1 and *C*2 in function form.

**5** The graph shows the function *y* = f(*x*) and two transformations of *y* = f(*x*), labelled *C*1 and *C*2.  
Write down the equations of the translated curves *C*1 and *C*2 in function form.

**6** The graph shows the function *y* = f(*x*).

**a** Sketch the graph of *y* = f(*x*) + 2

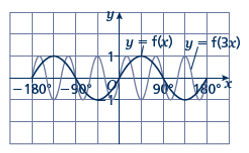
**b** Sketch the graph of *y* = f(*x* + 2)

**14b. Stretching graphs**

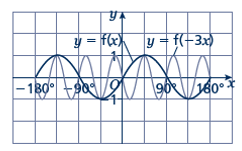
**A LEVEL LINKS**

**Scheme of work:** 1f. Transformations – transforming graphs – f(*x*) notation

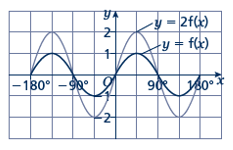
**Textbook:**Pure Year 1, 4.6 Stretching graphs

Key points

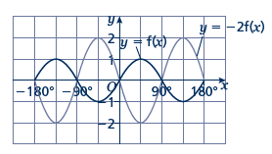
* The transformation *y* = f(*ax*) is a horizontal stretch of *y* = f(*x*) with scale factor  parallel to the *x*-axis.



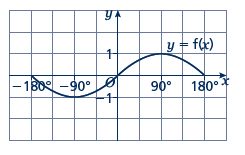
* The transformation *y* = f(–*ax*) is a horizontal stretch of *y* = f(*x*) with scale factor  parallel to the *x*-axis and then a reflection in the *y*-axis.



* The transformation *y* = *a*f(*x*) is a vertical stretch of *y* = f(*x*) with scale factor *a* parallel to the *y*-axis.



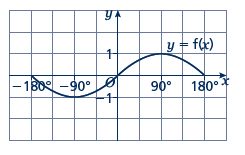
* The transformation *y* = –*a*f(*x*) is a vertical stretch of *y* = f(*x*) with scale factor *a* parallel to the *y*-axis and then a reflection in the *x*-axis.

Examples

**Example 3** The graph shows the function *y* = f(*x*).

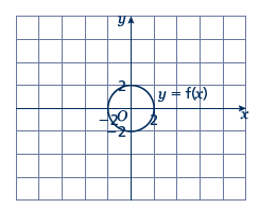
Sketch and label the graphs of   
 *y* = 2f(*x*) and *y* = –f(*x*).

|  |  |
| --- | --- |
|  | The function *y* = 2f(*x*) is a vertical stretch of *y* = f(*x*) with scale factor 2 parallel to the *y*-axis.  The function *y* = −f(*x*) is a reflection of *y* = f(*x*) in the  *x*-axis. |

**Example 4** The graph shows the function *y* = f(*x*).

Sketch and label the graphs of   
 *y* = f(2*x*) and *y* = f(–*x*).

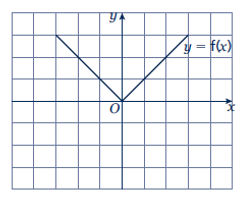
|  |  |
| --- | --- |
|  | The function *y* = f(2*x*) is a horizontal stretch of *y* = f(*x*) with scale factor  parallel to the *x*-axis.  The function *y* = f(−*x*) is a reflection of *y* = f(*x*) in the *y*-axis. |

Practice

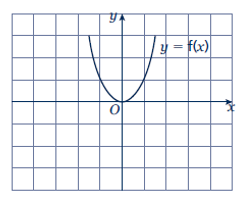
**7** The graph shows the function *y* = f(*x*).

**a** Copy the graph and on the same axes sketch and label the graph of *y* = 3f(*x*).

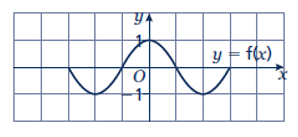
**b** Make another copy of the graph and on the same axes sketch and label the graph of *y* = f(2*x*).

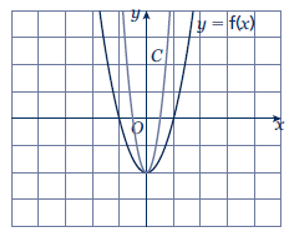


**8** The graph shows the function *y* = f(*x*).  
Copy the graph and on the same axes   
sketch and label the graphs of  
*y* = –2f(*x*) and *y* = f(3*x*).

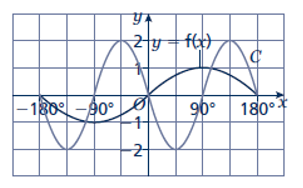


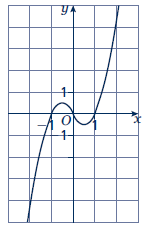
**9** The graph shows the function *y* = f(*x*).   
Copy the graph and, on the same axes,   
sketch and label the graphs of   
*y* = –f(*x*) and *y* =.

**10** The graph shows the function *y* = f(*x*).  
Copy the graph and, on the same axes,   
sketch the graph of *y* = –f(2*x*).



**11** The graph shows the function *y* = f(*x*) and a transformation, labelled *C*.  
Write down the equation of the translated curve *C* in function form.

**12** The graph shows the function *y* = f(*x*) and a transformation labelled *C*.  
Write down the equation of the translated curve *C* in function form.



**13** The graph shows the function *y* = f(*x*).

**a** Sketch the graph of *y* = −f(*x*).

**b** Sketch the graph of *y* = 2f(*x*).

Extend

**14 a** Sketch and label the graph of *y* = f(*x*), where f(*x*) = (*x* – 1)(*x* + 1).

**b** On the same axes, sketch and label the graphs of *y* = f(*x*) – 2 and *y* = f(*x* + 2).

**15** **a** Sketch and label the graph of *y* = f(*x*), where f(*x*) = –(*x* + 1)(*x* – 2).

**b** On the same axes, sketch and label the graph of *y* = .